RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College under University of Calcutta)

FIRST YEAR
B.A./B.SC. FIRST SEMESTER (July – December) 2014
Mid-Semester Examination, September 2014

Date : 15/09/2014 **MATHEMATICS (Honours)**
Time : 11 am – 1 pm Paper : I Full Marks : 50

[Use a separate answer book for each group]

Group – A

(*Answer **any five** questions*)

1. Determine the number of (i) antisymmetric relations, (ii) equivalence relations on the set {1,2,3,4} [5]

2. Let X be a nonempty set. A function $d : X \times X \to \mathbb{R}^+$ is defined to be a metric on X if
   a) d(x,y) = 0 for $x, y \in X \Leftrightarrow x = y$,
   b) d(x,y) = d(y,x) $\forall\ x, y \in X$
   and c) $d(x,z) \le d(x,y) + d(y,z)\ \forall\ x, y, z \in X$.
   Check if the map $D : \mathcal{P}(X) \times \mathcal{P}(X) \to \mathbb{R}^+$ defined by $D(A,B) = |A \Delta B|$ is a metric on $\mathcal{P}(X)$. [5]

3. Show that the number of even permutations on a finite set (containing at least 2 elements) is equal to the number of odd permutations on it. [5]

4. Let A, B be nonempty sets. Recall that a function $f : A \to B$ is invertible iff $\exists$ a map $g : B \to A$ such that gof = id_A, fog = id_B.
   Prove that a function $f : A \to B$ is invertible iff f is both left-invertible and right-invertible. [5]

5. Show that there exists no rational number r such that $r^2 = 5$. [5]

6. Find sup A and inf A if $A = \left\{ \frac{1}{m} + \frac{1}{n} : m, n \in \mathbb{N} \right\}$. [5]

7. a) Find all $A \subseteq \mathbb{R}$ such-that $\overline{A^o} = (\bar{A})^o$.
   b) Let $S \subseteq \mathbb{R}$. Show that int S is the largest open set contained in S. [5]

8. Find A' if $A = \left\{ (-1)^n \left(1 + \frac{1}{n}\right) : n \in \mathbb{N} \right\}$. [5]

Group – B

9. Answer **any two** questions :
   a) Show that the equation of the line joining the feet of perpendiculars from the point (d,0) on the lines : $ax^2 + 2hxy + by^2 = 0$ is (a–b)x + 2hy + bd = 0. [6]
   b) Find the polar equations of the directrices of the ellipse $\frac{\ell}{r} = 1 + e\cos\theta$. [6]
   c) If g be a variable tangent to the conic $\frac{\ell}{r} = 1 - e\cos\theta$, show that the locus of the foot of the perpendicular from the pole on g is the circle $r^2(e^2 - 1) + 2\ell e r\cos\theta + \ell^2 = 0$. [6]

10. Answer **any one** :
    a) Prove that the necessary and sufficient condition for $Mdx + Ndy = 0$ to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. [5]

b) Prove that if $Mx - Ny \neq 0$ then $\frac{1}{Mx - Ny}$ is an integrating factor of the equation $Mdx + Ndy = 0$ where $M = yf_1(xy)$, $N = xf_2(xy)$. [5]

11. Answer **any two** :

a) Solve : $(y^2 + 2x^2y)dx + (2x^3 - xy)dy = 0$ [4]

b) Find the general and singular solutions of $\sin\left(x\frac{dy}{dx}\right)\cos y = \cos\left(x\frac{dy}{dx}\right)\sin y + \frac{dy}{dx}$ [4]

c) Solve : $3x(1-x^2)y^2\frac{dy}{dx} + (2x^2 - 1)y^3 = ax^3$ [4]

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